Powers of Radicals

Assessment Type	**Recommended Grade Level**	**MYP Criterion Level**
Mathematical Investigation	Grade 10 (MYP5) Extended	MYP 5

MYP Assessment Criteria
Criterion B: Investigating patterns
Criterion C: Communicating

MYP Command Terms Used
Write down, calculate, find, describe, select, state, suggest, select, apply, verify, justify

MYP Global Context
Identities and relationships

MYP Key Concepts
Logic

MYP Related Concepts
Patterns, Generalization

MYP Branch of Mathematics
Numerical and abstract reasoning

MYP Topics and Skills

- Exponents and powers
- Squares and square roots
- Surds, roots, and radicals, including simplifying
- Laws of exponents, including integer and negative exponents

Prior Knowledge Needed

- Expanding and evaluating exponential expressions
- Writing expanded forms into exponential form
- Using calculator to evaluate radical and exponential expressions

Assessment Description

In this mathematical investigation, students explore patterns when radicals are raised to different powers (first even powers to help recognize patterns and then odd powers to check that the patterns they found actually works). Then, as they continue working with different radical expressions (raised to even and then odd powers), they discover the rules for writing radical expressions in exponential forms. In mark band 7-8, students expand their investigation into cube roots, 4th roots, and nth roots as well.

Materials Needed

Paper, pen, pencil, calculator

Task-specific instructions / Recommendations

Students are advised to follow the instructions throughout in order to recognize the necessary relationship/formula by the end of the investigation.

Assessment Criterion B: *Investigating patterns*

	Achievement Level Descriptor (MYP5)	Task Specific Descriptor
0	The student **does not** reach a standard described by any of the descriptors below.	
1-2	The student is able to: i. **apply**, **with teacher support**, mathematical problem-solving techniques to discover **simple patterns** ii. **state predictions** consistent with patterns iii. *(not demonstrated at this level)*.	The student is able to: i. **write down** the expanded forms, **calculate** the missing values, and **find** the missing expressions (Q1), (Q4) ii. **use** their previous findings to **state** their prediction for a rule that would help write a radical expression (Q6) iii. *(not demonstrated at this level).*
3-4	The student is able to: i. **apply** mathematical problem-solving techniques to discover **simple patterns** ii. **suggest general rules** consistent with **findings** iii. *(not demonstrated at this level)*.	The student is able to: i. **write down** the expanded forms, **calculate** the missing values, and **find** the missing expressions (Q7), (Q10) ii. **suggest** a general rule for writing $\sqrt{a}$ in exponential form (Q13) iii. *(not demonstrated at this level).*
5-6	The student is able to: i. **select** and **apply** mathematical problem-solving techniques to discover **complex patterns** ii. **describe patterns** as general rules consistent with **findings** iii. **verify** the validity of these general rules.	The student is able to: i. **select** and **apply** a method of their choice to further investigate the general rule (Q14) ii. **describe** the pattern found as a mathematical formula consistent with previous findings (Q14) iii. **verify** the validity of their rule (Q14)
7-8	The student is able to: i. **select** and **apply** mathematical problem-solving techniques to discover **complex patterns** ii. **describe patterns** as general rules consistent with **correct findings** iii. **prove**, or **verify** and **justify**, these general rules.	The student is able to: i. **select** and **apply** a method of their choice to expand the rule for the cube root of a number, and/or the 4^{th} root of a number, and the $n$the root of a number (Q15) ii. **describe** the pattern as a mathematical formula consistent with the formula suggested previously for square root (Q15) iii. **verify** and **justify** their rule for $\left(\sqrt[5]{3}\right)^4$ and two other examples (Q15)

Assessment Criterion C: *Communicating*

	Achievement Level Descriptor (MYP5)	Task Specific Descriptor
0	The student **does not** reach a standard described by any of the descriptors below.	
1-2	The student is able to: i. use **limited** mathematical language ii. use **limited forms** of mathematical representation to present information iii. *(not demonstrated at this level)* iv. communicate through lines of reasoning that are **difficult to interpret** v. *(not demonstrated at this level)*.	The student is able to: i. use mathematical vocabulary **poorly** ii. present his/her findings in an **unclear** or **limited** way iii. *(not demonstrated at this level)* iv. communicate his/her findings in a way that is **difficult to follow**. v. *(not demonstrated at this level).*
3-4	The student is able to: i. use **some appropriate** mathematical language ii. use **appropriate forms** of mathematical representation to present information **adequately** iii. *(not demonstrated at this level)* iv. communicate through lines of reasoning that are **complete** v. **adequately organize** information using a logical structure.	The student is able to: i. use mathematical vocabulary **somewhat clearly** ii. present his/her findings in a **somewhat clear** or **somewhat limited** way iii. *(not demonstrated at this level)* iv. communicate his/her findings **completely** v. communicate his/her findings in a way that is **somewhat organized**
5-6	The student is able to: i. **usually** use **appropriate** mathematical language ii. **usually** use **appropriate forms** of mathematical representation to present information **correctly** iii. **usually** move between different forms of mathematical representation iv. communicate through lines of reasoning that are **complete and coherent** v. **present** work that is **usually organized** using a logical structure.	The student is able to: i. use mathematical vocabulary **usually correctly** ii. present his/her findings **mostly clearly** and **correctly** iii. present his/her findings using **different** forms of representation iv. communicate his/her findings **completely** and **coherently** v. communicate his/her findings in a way that is **mostly organized**.
7-8	The student is able to: i. **consistently** use **appropriate** mathematical language ii. use **appropriate forms** of mathematical representation to **consistently** present information **correctly** iii. move **effectively** between different forms of mathematical representation iv. communicate through lines of reasoning that are **complete, coherent and concise** v. **present** work that is **consistently organized** using a logical structure.	The student is able to: i. use mathematical vocabulary **consistently correctly** ii. present his/her findings **consistently clearly** and **correctly** iii. present his/her findings **effectively** using different forms of representation iv. communicate his/her findings **completely**, **coherently,** and **concisely** v. communicate his/her findings in a way that is **organized** and follows a **logical** structure.

Introduction

The square root of a particular number is number that when multiplied by itself results in that particular number: For example, the square root of 49 is 7, because when 7 is multiplied by itself, the result is 49. More mathematically, we write this as $\sqrt{49} = 7$ because $7 \times 7 = 49$.

However, this begs the question: can we write the number $\sqrt{49}$ in exponential form instead of using the square root sign? Let's explore!

PART A: Exploring even powers

Let's start by exploring different powers of the number $\sqrt{2}$, starting with even powers first:

(1) In the table provided below,
   a. in the second column, **write down** the expanded form of the expression given in the first column,
   b. in the third column, **calculate** the value of the expression given in the first column,
   c. in the fourth column, **find** an expression in the form of 2 raised to a certain power.

   The first row has already been filled out for you.

[B: 1-2, i]

$\sqrt{2}$ raised to an **even** power	Expanded form	Value	2 raised to a power
$(\sqrt{2})^2$	$\sqrt{2}\times\sqrt{2}$	2	2^1
$(\sqrt{2})^4$	$\sqrt{2}\times\sqrt{2}\times\sqrt{2}\times\sqrt{2}$	4	2^2
$(\sqrt{2})^6$	$\sqrt{2}\times\sqrt{2}\times\sqrt{2}\times\sqrt{2}\times\sqrt{2}\times\sqrt{2}$	8	2^3
$(\sqrt{2})^8$	$\sqrt{2}\times\sqrt{2}\times\sqrt{2}\times\sqrt{2}\times\sqrt{2}\times\sqrt{2}\times\sqrt{2}\times\sqrt{2}$	16	2^4
$(\sqrt{2})^{10}$	$\sqrt{2}\times\sqrt{2}\times\sqrt{2}\times\sqrt{2}\times\sqrt{2}\times\sqrt{2}\times\sqrt{2}\times\sqrt{2}\times\sqrt{2}\times\sqrt{2}$	32	2^5

Table 1

(2) Briefly **describe** any relationship you notice between the expression given the first column and the expression you found in the last column.

The exponent given in the first column is twice the exponents found in the last column. Or, the other way around, the exponent found in the last column is half of the exponent given in the first column.

Let's continue exploring even powers, this time of the square root of a number other than 2:

(3) **Select** one of the numbers from the list below by circling your choice. *(You will use the square root of this number in the next question.)*

3 5 7 10 11

(4) Then, in the table provided below, after writing down your choice of number in the table header,
   a. In the first column, **write down** the number you chose in question (3) above,
   b. in the second column, **write down** the expanded form of the expression given in the first column,
   c. in the third column, **calculate** the value of the expression given in the first column,
   d. in the fourth column, **find** an expression in the form of the number you chose in question (3) raised to a certain power.

[B: 1-2, i]

$\sqrt{\square}$ raised to an **even** power	Expanded form	Value	$\square$ raised to a power
$\left(\sqrt{\square}\right)^{2}$	$\sqrt{\square}\times\sqrt{\square}$		
$\left(\sqrt{\square}\right)^{4}$			
$\left(\sqrt{\square}\right)^{6}$			
$\left(\sqrt{\square}\right)^{8}$			
$\left(\sqrt{\square}\right)^{10}$			

Table 2

****Use** your findings in questions (1) and (2) to help you with the power in the last column. Then, **use** your calculator to confirm that your answer is indeed correct.*

(5) Similarly to question (2), briefly **describe** any relationship you notice between the expression given the first column and the expression you found in the last column.

Regardless of the student's choice of number, the relationship described here should match what the student described in question (2) above.

(6) **Use** all your findings above and **state** your prediction,
   a. by **writing down** the missing value in the mathematical formula below, and
   b. by **writing down** your prediction with words

for a rule that would help us write a radical expression in exponential form.

[B: 1-2, ii]

$$\sqrt{a} = a^{\square}$$

Regardless of the student's choice of number, the student's prediction should be either 0.5 (if they choose to use decimals) or ½ (if they choose to use fractions).

PART B: Exploring odd powers

Next, let's continue exploring different powers of the number $\sqrt{2}$, this time with odd powers:

(7) In the table provided below,
   a. in the second column, **write down** the expanded form of the expression given in the first column,
   b. in the third column, **calculate** the value of the expression given in the first column, rounded to 3 decimal places,
   c. in the fourth column, **find** an expression in the form of 2 raised to a certain power.

[B: 3-4, i]

$\sqrt{2}$ raised to an **odd** power	Expanded form	Value *(rounded)*	2 raised to a power
$(\sqrt{2})^1$	$\sqrt{2}$	1.414	$2^{0.5}$
$(\sqrt{2})^3$	$\sqrt{2}\times\sqrt{2}\times\sqrt{2}$	2.828	$2^{1.5}$
$(\sqrt{2})^5$	$\sqrt{2}\times\sqrt{2}\times\sqrt{2}\times\sqrt{2}\times\sqrt{2}$	5.657	$2^{2.5}$
$(\sqrt{2})^7$	$\sqrt{2}\times\sqrt{2}\times\sqrt{2}\times\sqrt{2}\times\sqrt{2}\times\sqrt{2}\times\sqrt{2}$	11.314	$2^{3.5}$
$(\sqrt{2})^9$	$\sqrt{2}\times\sqrt{2}\times\sqrt{2}\times\sqrt{2}\times\sqrt{2}\times\sqrt{2}\times\sqrt{2}\times\sqrt{2}\times\sqrt{2}$	22.627	$2^{4.5}$

Table 3

****Use** your findings in questions (1), (2), (4), and (5) to help you with the power in the last column. Then, **use** your calculator to confirm that your answer is indeed correct.*

(8) Similarly to questions (2) and (5), briefly **describe** any relationship you notice between the expression given the first column and the expression you found in the last column.

The exponent given in the first column is twice the exponents found in the last column. Or, the other way around, the exponent found in the last column is half of the exponent given in the first column.

Let's continue exploring odd powers, this time of the square root of a number other than 2:

(9) **Select** one of the numbers from the list below by circling your choice. *(You will use the square root of this number in the next question.)*

3 5 7 10 11

(10) Then, in the table provided below, after writing down your choice of number in the table header,
   a. In the first column, **write down** the number you chose in question (9) above,
   b. in the second column, **write down** the expanded form of the expression given in the first column,
   c. in the third column, **calculate** the value of the expression given in the first column,
   d. in the fourth column, **find** an expression in the form of the number you chose in question (9) raised to a certain power.

[B: 3-4, i]

$\sqrt{\square}$ raised to an **odd** power	Expanded form	Value	$\square$ raised to a power
$\left(\sqrt{\square}\right)^{1}$	$\sqrt{\square}$		
$\left(\sqrt{\square}\right)^{3}$			
$\left(\sqrt{\square}\right)^{5}$			
$\left(\sqrt{\square}\right)^{7}$			
$\left(\sqrt{\square}\right)^{9}$			

Table 4

****Use*** *your findings in questions (1), (2), (4), (5), (7), and (8) to help you with the power in the last column. Then,* ***use*** *your calculator to confirm that your answer is indeed correct.*

(11) Similarly to questions (2), (5), and (8) briefly **describe** any relationship you notice between the expression given the first column and the expression you found in the last column.

Regardless of the student's choice of number, the relationship described here should match what the student described in questions (2), (5), and (8) above.

PART C: Putting it all together

In questions (2) and (5) you described patterns you recognized for even powers. Then, in questions (8) and (11) you described patterns you recognized for odd powers.

(12) Putting all of those together, briefly **describe** what all of those patterns have in common.

In all previous cases, the exponent given in the first column is twice the exponents found in the last column. Or, the other way around, the exponent found in the last column is half of the exponent given in the first column.

(13) Use your description in question (12) above to **suggest** a general rule for writing $\sqrt{a}$ in exponential form. *(Keep in mind: $\sqrt{a}$ is the same as $\left(\sqrt{a}\right)^1$.)*

[B: 3-4, ii]

$$\sqrt{a} = a^{1/2}$$

(14) **Select** and **apply** a method of your choice to further investigate the general rule you found and described in questions (1) – (12) above, this time with radical expressions of your choice (which should be different than what was used previously).

In your work, make sure to

a. **describe** the pattern you found as a mathematical formula consistent with what you suggested in question (13) above, and

b. **verify** the validity of your rule for $\left(\sqrt{7}\right)^4$ and two other (different) examples of your choice: one example of a radical expression raised to an <u>even</u> power and another example of a radical expression raised to an <u>odd</u> power
   i. by finding their values using a calculator, and
   ii. by applying the mathematical formula you suggested previously.

[B: 5-6, i-iii]

Use as many **forms of representation** as possible (tables, algebra, explanations, etc.) and **organize** your work in a **clear and logical manner**. Provide verification for your rule. Use scrap paper first if you need it.

Use the space below for your work.

Check student's individual investigation.

PART D: Expand the investigation to other roots

(15) **Select** and **apply** a method of your choice to expand the general rule for

a. the cube root of a number, and/or
b. the 4^{th} root of a number, and
c. the n^{th} root of a number.

In your work, make sure to

- **describe** the pattern you found as a mathematical formula consistent with the formula suggested previously for square roots, and
- **verify** and **justify** your rule for $\left(\sqrt[5]{3}\right)^4$ and two other (different) examples of your choice: one example of a fifth root radical expression raised to an <u>even</u> power and another example of a fifth root radical expression raised to an <u>odd</u> power,
  - by finding their values using a calculator, and
  - by applying the mathematical formula you suggested for the n^{th} root of a number.

[B: 7-8, i-iii]

Use as many **forms of representation** as possible (tables, algebra, explanations, etc.) and **organize** your work in a **clear and logical manner**. Provide verification and justification for your rule(s). Use scrap paper first if you need it.

Use the space below for your work.

Check student's individual investigation.